A SOLUTION TO THE PROJECTION PROBLEM FOR PRESUPPOSITION OF COMPOUND SENTENCES WITHIN ULRICH BLAU'S THREE-VALUED LOGIC SYSTEM

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In his work *Die dreiwertige Logik der Sprache* (1978), Ulrich Blau presents a semantically interpreted system of three-valued logic devised especially for the needs of natural language analysis. The ultimate semantic reasons of truthvaluelessness of sentences are semantic vagueness and reference failure occurring in clearly specified contexts. Within this system, presupposition can be non-vacuously defined.[1]

The definition comes close to the original Strawsonian one (Blau 1978:192):[2]

$$\text{F presupposes G in L3} \Leftrightarrow \forall\varphi\text{: } \varphi(\text{F})=1 \text{ or } 0 \Rightarrow \varphi(\text{G})=1$$

The definition says that a formula F presupposes a formula G in Blau's three-valued logic system of and only of, of any φ function (any semantic interpretation function) assigns the truth-value: true or false to F, then it follows that the same function assigns the truth-value: true to G. In other words, whenever F is true or false, then its presupposition G is true. The definition is not circular because truth-values: true, false, truthvalueless are assigned to formulae independently of the notion of presupposition.

Among other logical connectives, U. Blau's system contains the following four one-place logical operators:[3]

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[1] It seems that a non-vacuous definition of presupposition must be a part of a semantic theory within which truthvaluelessness of sentences is predicted independently of presupposition.

[2] U. Blau gives two other equivalent versions of this definition.

[3] The operators are not basic within Blau's system. They can be defined in terms of the basic ones which are $\frown$ — strong negation, $\urcorner$ — weak negation, $\wedge$ — conjunction. Consequently, the occurrence of $\perp$, $\top$, $+$, ⌶ in the formulae introduced later in the paper could be eliminated in favour of the basic operators. This, however, would lead to the loss of the distinctions (among (i), (ii), and (iii)) introduced on purpose.

$\top$ — one-place truth operator corresponding to the natural language predicate *'it is true that S'*

$\perp$ — one-place falsity operator corresponding to the natural language predicate *'it is false that S'*

$+$ — one-place indetermination operator corresponding to the natural language predicate *'it is neither true nor false that S'*

$\underline{\top}$ — one-place determination operator corresponding to the natural language predicate *'it is either true or false that S'*

The semantic interpretation of $\top$, $\perp$, $+$, $\underline{\top}$ is given by the following truth-tables:

F	$\top$F	$\perp$F	$+$F	$\underline{\top}$F
1	1	0	0	1
0	0	1	0	1
$\frac{1}{2}$	0	0	1	0

The most important is the truth operator $\top$; when a sentence S is true or false, then 'it is true that S' is correspondingly true or false. When S is truthvalueless, then 'it is true that S' is not something indeterminate but false.

Those operators, together with the truth-tables for conjunction and alternative[4] allow to specify what presuppositions of compound (conjunctive and disjunctive) sentences are.

The projection problem for presupposition of compound sentences, i.e. sentences with conjunctions 'and' and 'or' (and possibly 'if... then') has not been successfully solved within Frege's or Karttunen's frameworks.[5] Blau

[4] As well as with two negation operators: strong negation

p	$-$p
0	1
1	0
$\frac{1}{2}$	$\frac{1}{2}$

and weak negation

p	$\neg$ p
0	1
1	0
$\frac{1}{2}$	1

whose distributions are specified. The negation present in the examples to come is to be understood as strong negation.

[5] For criticism see, for example Wilson (1975) or Soames (1976). One of the most extensive discussions of the problem of presupposition of compound (disjunctive and conditional) sentences is presented in S. Soames's (1979) article "A Projection Problem for Speaker Presuppositions". The article contains a critique of the solution to the problem offered by three-valued accounts of logical presupopsition, as well. Yet, the truthtables for conjunction and implication discussed by Soames differ from Blau's truth-tables.

The aim of the present paper is to describe a solution to the projection problem deducible from U. Blau's system and not to reexamine Soames's arguements in detail. But since the two problems overlap, the examples to follows come (predominantly) from Soames's paper. The interpretation of the examples is different.

does not suggest the solution of this problem, either. Yet, he provides the truth-tables for 'and' and 'or'. It is possible to deduce what presuppositions of compound sentences with those conjunctions are basing on the information provided in the truth-tables.

By definition, presupposition is a condition that must be satisfied (i.e. the sentence expressing the condition must be true) for a sentence to be true or false. If the condition fails, the sentence is neither true nor false.

From the truth-table for the conjunction 'and' it follows that a compound sentence containing this conjunction is truthvalueless if either both conjuncts are neither true nor false or one of them is truthvalueless and the other is true:

p \ q	1	0	$\frac{1}{2}$
1	1	0	$\frac{1}{2}$
0	0	0	0
$\frac{1}{2}$	$\frac{1}{2}$	0	$\frac{1}{2}$

Consequently, the presupposition of a conjunctive sentence is the condition that the situation described above cannot occur, i.e. if 'p and q' is to be true or false it is impossible for the presuppositions of both p and q to be not true or for the presupposition of one of them (p or q) to be not true with the other conjunct being true at the same time.

It is possible to express what a presupposition of a conjunctive sentence is more formally with the help of the operators introduced earlier.

Let p, q be the component sentences of a conjunctive compound sentence and let p^p, q^p be the respective presuppositions of p and q established by the rules which attach presuppositions to simple and complex sentences. 'p and q' is the conjunctive sentence itself. Then $(p \text{ and } q)^p$ is the presupposition of a whole conjunctive sentence, which may be defined in the following way:

$$
\begin{array}{lll}
(p \text{ and } q)^p = & \text{(i)} & \top p^p \wedge \top q^p \\
\text{or} & \text{(ii)} & (+p^p \vee \perp p^p) \wedge \top q^p \wedge \perp q \\
\text{or} & \text{(iii)} & (+q^p \vee \perp q^p) \wedge \top p^p \wedge \perp p
\end{array}
$$
[6]

This may be paraphrased in the following way: presupposition of a conjunctive sentence expresses the condition that either (i) the presuppositions of both conjuncts are true; or (ii) the presupposition of p fails and q is false; or (iii) the presupposition of q fails and p is false. If none of (i)—(iii) is the case, then the

[6] (ii) and (iii) are equivalent to (ii) '$\neg \top p^p \wedge \perp q$, (iii) '$\neg \top q^p \vee \perp p$, respectively. (ii) and (iii), however, bring out the relation between the presuppositions of separate conjuncts and those of the whole conjunction in a more explicit way.

sentence of the form 'p and q' is neither true nor false. It follows that if p and q share the same presupposition, then it becomes a presupposition of the whole conjunction, and of it fails^p'p and q' becomes neither true nor false.

The presupposition of a conjunctive sentence is thus a very general (although clearly specified) condition imposed on the possible co-occurrence of the failure of presupposition of one constituent sentence and specific truth evaluationof the other constituent sentence.[7] Which of the admitted co-occurrences is the case when a given sentence is uttered is a matter specified by pragmatics.

In agreement with the definition, the sentences below are assigned the truth-values given in parentheses:

1. All of Jack's children are bald and all of his children are naughty (assumption: Jack is childless) ($\frac{1}{2}$)
2. The king of France is bald and France is a monarchy (0)
3. Ruritania exists and its ruler is very powerful (0)
4. The king of France is bald and France is a republic ($\frac{1}{2}$)
5. Ruritania is an imaginary country and its ruler is very powerful ($\frac{1}{2}$)
6. All of Jack's children are bald and Jack has children (assumption: Jack is childless) (0)
7. Jack has children and his sons are bald
   (assumption 1: Jack has two daughters only) ($\frac{1}{2}$)
   (assumption 2: Jack is childless) (0)
8. The lawyer located Susan's only heir and he found the oldest of her heirs
   (assumption 1: Susan has only one heir, and the lawyer didn't find him; or — Susan has more than one heir, and the lawyer didn't find the oldest one) (0)
   (assumption 2: Susan has only one heir, and the lawyer located him; or — Susan has more than one heir, and the lawyer found the oldest one) ($\frac{1}{2}$)
   (assumption 3: Susan has no heirs) ($\frac{1}{2}$)

(1) illustrates a situation in which two conjuncts share the same presupposition which, then, becomes a presupposition of the whole sentence. If it fails, the sentence is neither true nor false.

(2)/(3) show that if the presupposition of one conjunct fails but the other sentence is false, the presupposition of the whole sentence is preserved in agreement with (ii) and (iii) and the sentences are assigned the truth-value: false, as predicated by the truth-table for conjunction.

If, however, the presupposition of one conjunct fails and the other conjunct is true, then none of the conditions specified under (i)-(iii) is fulfilled, and sentences (4)/(5) are assigned the truth-value: neither true nor false because of

[7] It is possible to speak about two sentences, only, because any conjunction can be interpreted as consisting of two conjuncts.

presupposition failure, which follows from the truth-table for conjunction.

(6) exemplifies a situation in which one conjunct expresses the presupposition of the other. If this presupposition fails, the whole sentence is false and not truthvalueless as was counterintuitively predicted by the cumulative hypothesis.

(7) shows that the requirement that the conjunct whose presupposition is true (of the presupposition of the other conjunct fails) must be false forms a part of presupposition of the whole conjunction.

(8) presents the situation in which the presuppositions of two conjuncts are incompatible.

The assignment of truth-values to (1)-(8) seems to be supported by intuitions.

In summary, sentences of the form 'p and q' presuppose what at least one conjunct presupposes. If presupposition of only one conjunct is true, then the requirement that this conjunct be false constitutes a part of presupposition (=condition for the truth or falsity) of the whole conjunction.

In a very similar way presupposition of sentences of the form 'p or q' may be described.

From the truth-table for the connective 'or' it follows that a compound sentence containing this connective is truthvalueless of either both disjuncts are neither true nor false or one of them is truthvalueless with the other being false at the same time:

p \ q	1	0	$\frac{1}{2}$
1	1	1	1
0	1	0	$\frac{1}{2}$
$\frac{1}{2}$	1	$\frac{1}{2}$	$\frac{1}{2}$

Consequently, the presupposition of a disjunctive sentence is the condition that the situation described above cannot occur, i.e. if 'p or q' is to be true or false, it is impossible for the presuppositions of both p and q to be false or for the presupposition of one of them to be false with the other disjunct being false at the same time. More formally:

$$
\begin{aligned}
(p \text{ or } q)^p = & \text{(i)} \top p^p \wedge \top q^p \\
& \text{or (ii) } (+p^p \vee \bot p^p) \wedge \top q^p \wedge \top q \\
& \text{or (iii) } (+q^p \vee \bot q^p) \wedge \top p^p \wedge \top p^8
\end{aligned}
$$

[8] (ii) and (iii) are equivalent to (ii) '$\neg \top p^p \wedge \top q$, (iii) '$\top q^p \wedge \top p$, respectively. (ii) and (iii), however, bring out the relation between the presuppositions of component sentences and those of the whole disjunction in a more explicit way. (I)-(iii) basically correspond to the presupposition of alternative specified as $(p \vee q^p) \& (q \vee p^p)$ and criticized by Soames (1979: 635).

This may be paraphrased as: presupposition of a disjunctive sentence expresses the condition that either (i) the presuppositions of both disjuncts are true or (ii) the presupposition of p fails and q is true; or (iii) the presupposition of q fails and p is true. If none of (i)-(iii) is the case, then the sentence of the form; 'p or q' is neither true nor false. It follows that of p and q share the same presupposition, then it becomes a presupposition of whole disjunction, and if it fails 'p or q' becomes neither true nor false.

The analysis of several sentences[9] will show what kind of predictions follow from defining the presupposition of disjunctive sentences in the way presented above.

9. Either the chairman of the department isn't a skeptic about the external world or the chairman of the department isn't very sensible≫The department has a chairman
10. Either the king of France isn't wise, or the French generals aren't under the king's control≫There is a king of France
11. Either every radical in this room will escape, or every one of them will die≫There is at least one radical in the room
12. Either all of John's children are asleep or none of his children are asleep≫John has at least one child
13. Either there is no king of France or the king of France is in hiding (1)
14. Either all of Jack's letters have been held up or he has not written any (1)
    (assumption: Jack has written no letters)
15. Either all of Jack's children are asleep or Jack is a castrato ($\frac{1}{2}$)
    (assumption: Jack is childless but not a castrato)
16. Either the king of France is bald or no bald people exist ($\frac{1}{2}$)
17. Either the loan company repossessed Bill's only car, or they repossessed his second car
    (assumption 1: Bill had only one car and the loan company repossessed it or Bill had at least two cars and the loan company repossessed the second car) (1)
    (assumption 2: Bill had only one car and the loan company did not repossess it or Bill had at least two cars and the loan company did not repossess his second car) ($\frac{1}{2}$)
    (assumption 3: Bill had no car) ($\frac{1}{2}$)
18. Either the lawyer located Susan's only heir or his assistant located the oldest of her several heirs.

Examples (9)/(12) show what happens if two disjuncts have the same presupposition. Then this presupposition becomes a presupposition of the whole disjunctive sentence, and of it fails the sentence is neither true nor false. This follows from (i)-(iii) and from the truth-table for disjunction.

(13)/(14) illustrate the situation in which one disjunct expresses the negation of the presupposition of the other disjunct. If this negation is true, then, naturally, the other disjunct is neither true nor false because of presupposition failure, but the whole compound sentence is true as predicted by (ii) and (iii) and by the truth-table for alternative.

In (15)/(16), the presupposition of one disjunct fails and the other disjunct is false. Consequently, the whole sentence is neither true nor false. This shows that the requirement that the disjunct whose presupposition is true (if the presupposition of the other disjunct fails) must be true forms a part of presupposition of the whole disjunction.

The presuppositions of the two disjuncts in each of (17) and (18) are incompatible. The presupposition of the whole compound sentence is that the presupposition of at least one disjunct must be true with this disjunct being true at the same time. Which of the disjuncts is true is left undecided. Disjunctive sentences with incompatible presuppositions may be true but not false. It follows from the fact that the presuppositions of two disjuncts cannot be true together, so at least one disjunct must be neither true nor false, which prevents the whole disjunction from being false.

(18) requires additional comments. Intuitively, the sentence implies that the lawyer has an assistant and those intuitions should be accounted for. The presupposition of (18) cannot be expressed by (i) because the two disjuncts have incompatible presuppositions. If (iii) is the case, then the second disjunct is true and since it presupposes that the lawyer has an assistant, the implication is not lost. If, however, (ii) is the case, the implication is lost. It follows that the definition of (p or q) T^p allows but does not require "peripheral" presuppositions of particular disjuncts to become parts of the presupposition of a whole sentence. This is an inadequacy of the definition. In all other cases the predictions about (9)-(17) seem to be supported by intuitions.

In summary, sentences of the form 'p or q' presuppose what at least one disjunct presupposes. If presupposition of only one disjunct is true, then the requirement that this disjunct be true constitutes a part of pressupposition of the whole disjunction.

The problem of presuppositions of sentences containing the 'if...then' conjunction is more complicated. Such sentences, called conditionals, are syntactically complex and not compound — the *if-* clause being subordinate to the other clause. The correspondence between natural language conditional sentences and the logical connective of implication is definitely not one to one correspondence. It may even be justified to question the existence of such a correspondence as far as at least some conditional sentences are concerned. The question is to what extent can the 'if ... then' natural language conjunction be

[9] Examples (9)-(13) and (17), (18) come from Soames (1979). The distinction between the inclusive and exclusive 'or' was not made there.

treated as truth-functional, and as such allowing to speak about presuppositions or entailments carried by sentences of the form 'if p then q'.

U. Blau's standpoint (1978:91, 92) is that conditional sentences should not be formalized with the help of his three-valued implication. This logical connective is used for the formalization of sentences with the universal quantifier: All [F] are [G].

The only truth-functional characteristic of natural language conditionals is that if the antecedent is true and the consequent is false, then the whole conditional sentences is false, as in

19. If the king of France doesn't exist, then France is a monarchy

If Blau's standpoint is accepted, then no genuine presuppositions of conditionals exist. Various inferences deducible from conditional sentences should then be drawn in nontruth-conditional terms, ie in terms of implicatures and implications of various types.

It is possible to classify conditionals into various groups, some of which may be claimed to be analysable in truth-conditional terms. One of such classifications is presented by J. Schachter (1971), who also examines the possible presuppositions carried by some conditional sentences.

It seems, however, that no general agreement as far as the theory of conditionals is concerned has been reached. Yet, such a theory is a precondition of an adequate presuppositional analysis of such sentences.

Nevertheless, many linguists, for various purposes, do treat conditional sentences as corresponding to some kind of logical implication. It is interesting to notice that if presuppositions of conditionals were computed from the truth-table for U. Blau's three-valued implication, then at least some of the intuitions would be correctly reflected.

The truth-table for this implication is

p \ q	$p \to q$: 1	0	$\frac{1}{2}$
1	1	0	$\frac{1}{2}$
0	1	1	1
$\frac{1}{2}$	1	1	1

It follows that the implication is truthvalueless only if the antecedent is true and the consequent is neither true nor false. Consequently, the presupposition of an implication is

$$(p \to q)^p = \top p \to \top q^p$$

since only of it is not the case that true p implies that the presupposition of q is true, the whole implication 'p → q' is truthvalueless.

From this it follows that if[10]

20. If Bill didn't meet the king of Slobovia, he met the president of Slobovia

is to be true or false, then if it is true that Bill didn't meet the king of Slobovia, then it must be true that Slobovia has a president. This does not mean, however, that (20) presupposes the existence of the king of Slobovia or of the president of Slobovia. Similarly, if

21. If the lawyer didn't locate Susan's only heir, he located the oldest of her (several) heirs

is to be true or false, it follows that if it is true that the lawyer didn't locate Susan's only heir, then it must be true that she has several heirs. This is not equivalent to the claim that Susan has one or more heirs. Also, sentences (22), (23) do not presuppose (22a), (23a) but (22b), (23b), which agrees with intuitions.

22. If there is a king of France, then the king of France is in hiding
22a. The king of France exists
22b. If it is true that the king of France exists, then it must be true that France has a king
23. If John has children, then all of his children are intelligent
23a. John has children
23b. If it is true that John has children, then it must be true that he is not childless[11]

The analysis of conditional sentences constitutes a separate issue. Only an adequate theory of such sentences may be a starting point for the proper investigation of the problem of their possible presuppositions.

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[10] Examples (20)-(23) come from Soames (1979).

[11] The tautologous character of (22b), (23b), follows from the fact that the antecedents of (22), (23) express the presuppositions of the respective consequents.

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